ROLE OF COMPLEX MODES IN MODELING DISCONTINUITIES OF DIELECTRIC LOADED WAVE GUIDES *

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ABSTRACT

It is shown that modeling of step discontinuities in cylindrical dielectric loaded waveguides excited in hybrid modes, using mode matching cannot converge unless complex modes are included in the field expansions. If the parameters of the structure and operating frequency allow the existence of complex modes, then the purely propagating and purely evanescent mode fields are not a complete set, unless complemented by the complex mode fields.

Numerical results are presented that clearly illustrate the role of the complex mode fields in step discontinuity modeling.

1. INTRODUCTION

It has been known [1]-[3] that inhomogeneously filled waveguides can support, in addition to the evanescent and the propagating modes, complex modes, characterized by complex propagation constants. Recently, generalized rigorous analysis of lossless inhomogeneously filled waveguides [4], and numerical methods for the investigation of their properties [5] have been presented, which derived many important properties of complex modes. Generally, numerical search for the propagation constants of complex modes is a much more difficult problem than for the normal propagating and evanescent modes. Complex modes usually exist for very limited ranges of structure parameters and frequencies. When solving for discontinuity problems in guiding structures, which could support complex modes one is always faced with the problem of whether it is necessary to consider and include complex modes in the solution. This question has been recently addressed for the problem of finline discontinuities [6], where it was shown that ignoring a complex mode results in violation of complex power conservation across the discontinuity.

This paper analyzes the step discontinuity problem in a dielectric loaded waveguide, and the role of complex modes in the solution of such a problem. The general structure under consideration is shown in Figure 1. It consists of two semi-infinite circular dielectric loaded waveguides of different cross sectional dimensions joined at the plane z=0. A single hybrid mode of unit amplitude (HE_{mn}) is incident from $z < 0$ on the discontinuity. It is desired to determine the amplitudes of all the reflected and transmitted hybrid modes. In particular, if complex modes can exist in either or both waveguides, and these complex modes are included or excluded from the solution, what are the difference in the resulting scattering matrices?

Before presenting the solution using mode matching, the properties of complex modes on cylindrical dielectric loaded waveguides are summerized and typical numerical data on these modes are presented.

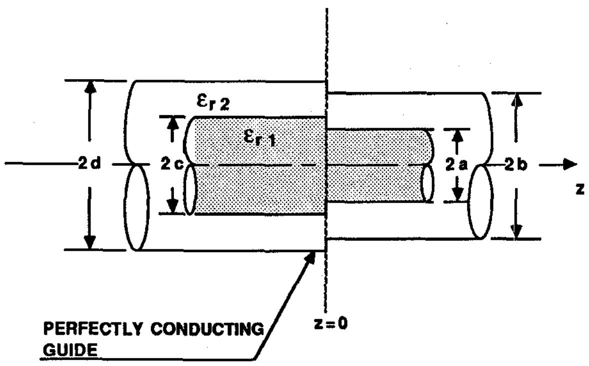

Fig.1 Step discontinuity at the junction between two semi-infinite dielectric loaded waveguides

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2. COMPLEX MODES ON DIELECTRIC LOADED WAVEGUIDES

Complex modes are characterized by a complex propagation constant γ which is obtained by searching for complex roots of the characteristic equation [7] for hybrid modes. Since the characteristic equation is real even function of γ, its complex roots occur in conjugate pairs. Further, if γ is a root, then $(-\gamma)$ is also a root, it therefore results that there is always a quadruple of complex roots: $\pm\gamma$ and $\pm\gamma^*$. In an infinite guide with no sources at infinity, a combination of a pair of complex conjugate modes must always be present. This pair will only carry reactive power, no net real average power. Typical $(\omega - \beta)$ diagrams for two dielectric loaded waveguides with parameters that allow complex modes to be present are shown in Figs. 2 and 3. In these figures, the propagation constant $\gamma = \alpha + j\beta$ of the hybrid modes with angular variation $e^{j\phi}$ in the dielectric loaded waveguide are plotted versus frequency. The solid curve is either purely real attenuation constant (αa) or purely imaginary propagation constant (βa). The dotted curves are the complex propagation constant γa. Complex propagation in Fig. 3 occurs in the frequency ranges from 1.7 GHz $\leq$ f $\leq$ 3.01 GHz and 5.22 GHz $\leq$ f $\leq$ 5.64 GHz for the hybrid modes (HE_{11}, HE_{12}) and (HE_{15}, HE_{16}) respectively. It has been shown [4] that the complex modes are linearly independent from all other propagating and evanescent modes occurring at the same frequency. Therefore to expand any arbitrary field over the cross section of the dielectric loaded waveguide at any frequency requires the inclusion of any complex modes that may exist. All the modes have been shown to be orthogonal to each other over the cross section of the guide.

The above properties are used in the following section to obtain a model for the step discontinuities of the dielectric loaded waveguide shown in Fig. 1 using the mode matching technique.

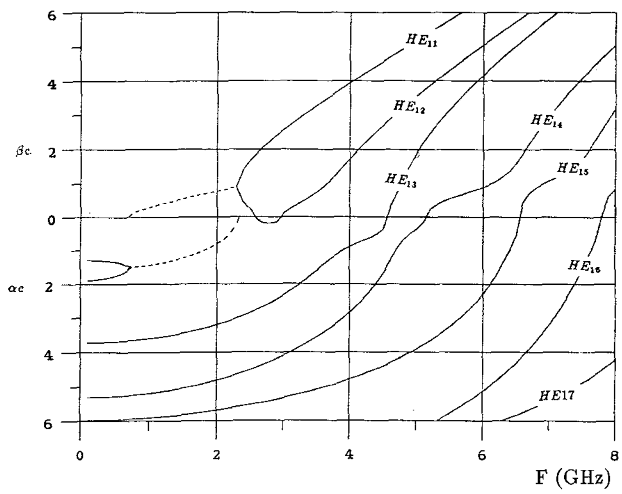

Fig. 2 $(\omega - \beta)$ diagram for the region $z < 0$.

$c = 0.35$
$d = 0.5$
$\epsilon_{r_1} = 37.$
$\epsilon_{r_2} = 1.$

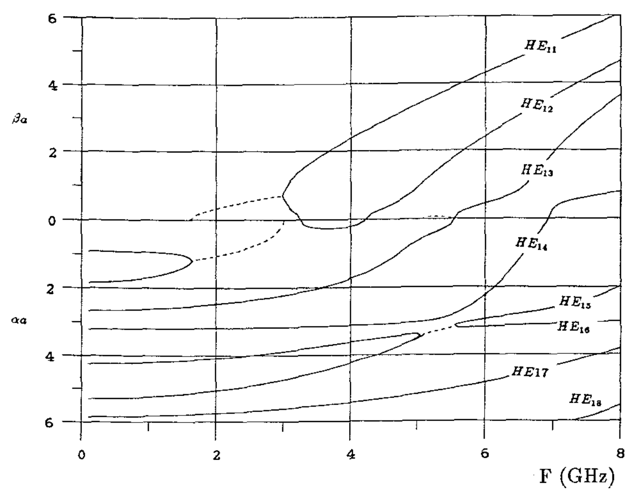

Fig. 3 $(\omega - \beta)$ diagram for the region $z > 0$.

$a = 0.25$
$b = 0.5$
$\epsilon_{r_1} = 37.$
$\epsilon_{r_2} = 1.$

3. DISCONTINUITY CHARACTERIZATION

Consider a hybrid HE_{mn} mode incident from $z < 0$ on the step discontinuity of the two semi infinite dielectric loaded waveguides shown in Fig. 1. Due to this discontinuity reflected and transmitted fields will be generated in the regions $z < 0$ and $z > 0$ respectively. All the fields will have the same azimuthal variation $(e^{jm\phi})$ as the incident hybrid mode. In order to calculate the reflected and transmitted fields the total transverse fields are expanded in terms of the appropriate hybrid waveguide modes on both sides of the discontinuity. Thus:

For $z < 0$.

$$\bar{E}_t = \hat{e}_{A1}\, e^{-\gamma_{A1} z} + \sum_j A_j\, \hat{e}_{Aj}\, e^{\gamma_{Aj} z} \qquad (1-a)$$

$$\bar{H}_t = \hat{h}_{A1}\, e^{-\gamma_{A1} z} - \sum_j A_j\, \hat{h}_{Aj}\, e^{\gamma_{Aj} z} \qquad (1-b)$$

For $z > 0$.

$$\bar{E}_t = \sum_k B_k \, \hat{e}_{Bk} \, e^{-\gamma_{Bk} z} \qquad (2-a)$$

$$\bar{H}_t = \sum_k B_k \, \hat{h}_{Bk} \, e^{-\gamma_{Bk} z} \qquad (2-b)$$

where $\hat{e}_{Aj}$, $\hat{h}_{Aj}$, γ_{Aj}, $\hat{e}_{Bk}$, $\hat{h}_{Bk}$, γ_{Bk} are the transverse electric and magnetic fields and the propagation constants of the hybrid modes in the regions $z < 0$ and $z > 0$ respectively.

Applying the boundary conditions at z=0 that the tangential electric and magnetic fields are continuous and using the orthogonality relationships of the hybrid modes, an infinite system of equations can be obtained in which the unknowns are the expansion coefficients A_j and B_k. Numerical solution of this system is achieved by truncating the infinite matrix and solving the resulting finite system of linear equations.

If the modes included in equations 1 and 2 are complete sets (which must include any complex modes), then the solution for the reflected and transmitted field coefficients will always converge to the correct answer. If the complex modes are not included the solution may not converge or may converge to the wrong answer.

A numerical example illustrating the above analysis is presented. The $(\omega - \beta)$ diagram of the waveguides in the region $z < 0$ and $z > 0$ are shown in Fig. 2 and Fig. 3 respectively. The solutions are obtained with and without the inclusion of the complex modes. To check the validity of the solution, the total fields were computed from the coefficients of expansion and the boundary conditions on these fields at z=0 are verified. A quantitative measure of the error in satisfying the boundary conditions in the electric and magnetic field components used in the computation is defined by:

$$\epsilon = 4 \frac{\int_S |(field\ component\ at\ z = 0^+) \ - \ (field\ component\ at\ z = 0^-)|^2 \ dS}{\int_S |(field\ component\ at\ z = 0^+) \ + \ (field\ component\ at\ z = 0^+)|^2 \ dS}$$

where S is the guide cross section. It was found that when the complex modes are not included, the boundary conditions at z=0 are not satisfied regardless of the number of modes included in the solution as shown in Fig. 4. On the other hand, when the complex modes are included in the expansion, the boundary conditions are satisfied and the error in the boundary conditions decreases as the number of modes is increased as shown in Fig. 5.a and Fig. 5.b.

Complete scattering matrix which characterizes the step discontinuity is easily obtainable from the results of the above analysis. Typical result showing the variation of the scattering matrix of a discontinuity is shown in Fig. 6.

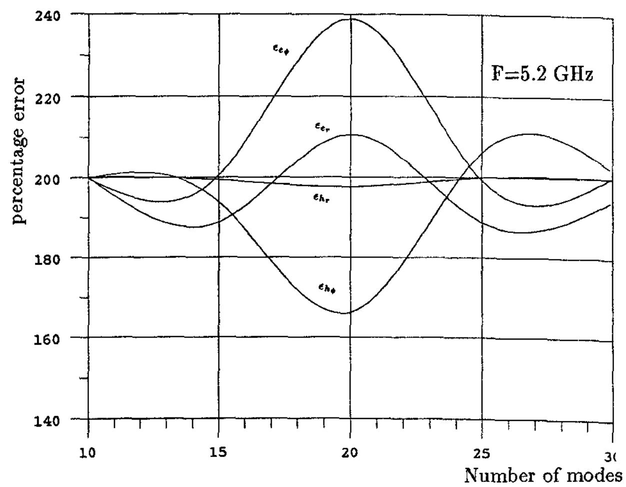

Fig4. Variation of the percentage error in the field intensity with number of modes without complex modes.

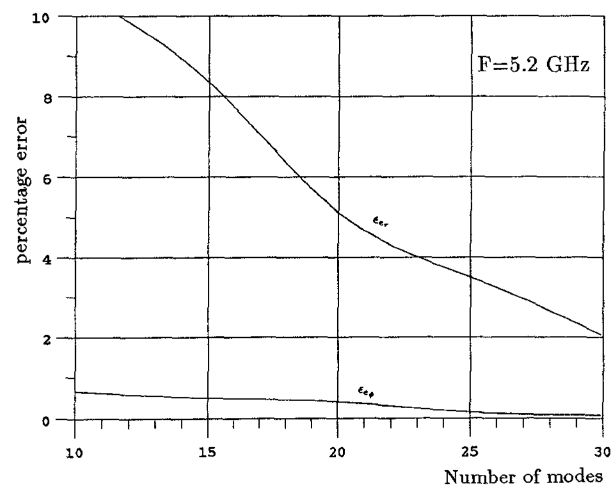

Fig. 5.a Variation of the percentage error in field intensity with number of modes when complex modes are included.

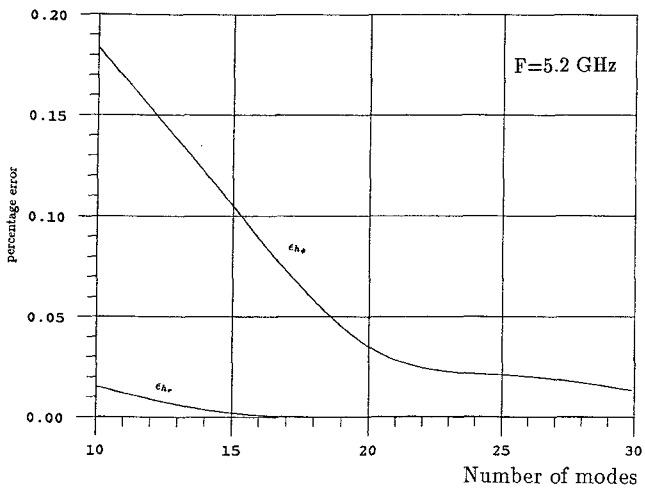

Fig. 5.b Variation of the percentage error in field intensity with number of modes when complex modes are included.

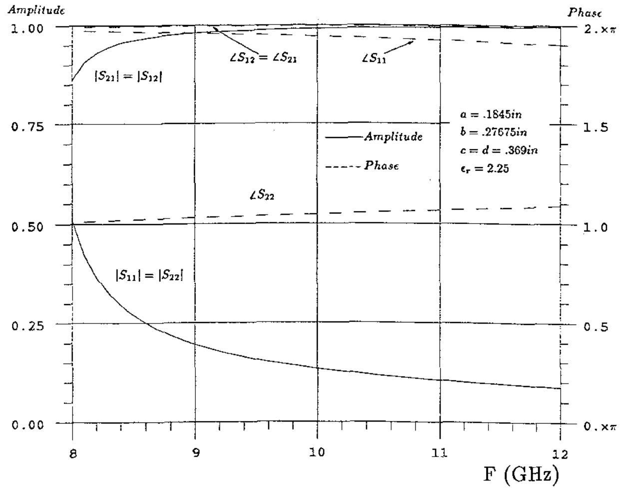

Fig. 6 Variation of the scattering parameters vs. frequency.

4. CONCLUSION

It is shown through numerical calculations of specific examples that complex modes are part of a complete set that represent the total fields in dielectric loaded waveguides. Solution of the step discontinuity problem in a dielectric loaded waveguide has been obtained using mode matching and verification of the accuracy and convergence of the solution has been presented. A circuit model for the step discontinuity in a hybrid mode dielectric loaded waveguide will be presented.

Although the examples presented are in the microwave region, and for the HE_{11}-mode, this type of transmission medium is useful for millimeter, submillimeter, and optical wavelengths. In order to reduce the transmission loss, higher order modes may be used. The technique presented for the discontinuity characterization is general and applicable for any mode and frequency band.

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